PHIL 120A – 2nd Exam – Fri. July 21, 2006 – 100 points

1) **(26 points)** Using Lemmon's system of rules, prove the following sequent without using the contamination maneuver:
P & -Q ├ - (-P v Q)

1	(1)	P & -Q	A
2	(2)	-P v Q	A / RAA
3	(3)	-P	A / vE
1	(4)	P	1 &E
1,3	(5)	P & -P	3,4 &I
~~1~~,3	(6)	-(P & -Q)	1,5 RAA
7	(7)	Q	A / vE
1	(8)	-Q	1 &E
1,7	(9)	Q & -Q	7,8 &I
~~1~~,7	(10)	-(P & -Q)	1,9 RAA
2,~~3,7~~	(11)	-(P & -Q)	2,3,6,7,10 vE
1,2	(12)	(P & -Q) & -(P & -Q)	1,11 &I
1,~~2~~	(13)	- (-P v Q)	2,12 RAA

(Assumption 1 is discharged on lines 6 and 11)
(Assumptions 3 and 7 are discharged on line 11)
(Assumption 2 is discharged on line 13)

2) (**14 points**) Using Lemmon's system of rules, prove the following sequent without using the contamination maneuver:
P ↔ - Q ├ Q → -P

1	(1)	P ↔ - Q	A
1	(2)	(P→ -Q) & (-Q → P)	1 Df. ↔
1	(3)	P→ -Q	2 &E
4	(4)	Q	A / CP ├ -P
4	(5)	- - Q	4 DN
1,4	(6)	-P	3,5 MTT
1,**4**	(7)	Q → -P	4,6 CP

(Assumption 4 is discharged on line 7)

3) (**20 points**) Using Lemmon's system of rules, prove the following sequent:

⊢ -P → (P → Q)

1	(1)	-P	A / CP ⊢ P → Q
2	(2)	P	A / CP ⊢ Q
3	(3)	-Q	A / RAA
2,3	(4)	P & -Q	2,3 &I
2,3	(5)	P	4 &E
1,2,3	(6)	P & -P	1,5 &I
1,2,~~3~~	(7)	- - Q	3,6 RAA
1,2	(8)	Q	7 DN
1,~~2~~	(9)	P → Q	2,8 CP
~~1~~	(10)	-P → (P → Q)	1,9 CP

(Assumption 3 is discharged on line 7)
(Assumption 2 is discharged on line 9)
(Assumption 1 is discharged on line 10)

4) (**22 points**) Using Lemmon's system of rules, prove one or the other of the following sequents (your choice):

P v - Q , Q ⊢ P

- (P → -Q) ⊢ P & Q
(See page 4 below)

1	(1)	P v -Q	A
2	(2)	Q	A
3	(3)	P	A / vE
4	(4)	-Q	A / vE
5	(5)	-P	A / RAA
2,5	(6)	Q & -P	2,5 &I
2,5	(7)	Q	6 &E
2,4,5	(8)	Q & -Q	4,7 &I
2,4,~~5~~	(9)	- - P	5,8 RAA
2,4	(10)	P	9 DN
1,2,~~3,4~~	(11)	P	1,3,3,4,10 vE

(Assumption 5 is discharged on line 9)
(Assumptions 3 and 4 are discharged on line 11)

5) (9 points) Carefully examine this attempted proof and answer the questions below.

1	(1)	- (P → -P)	A	Assumption 2 is discharged on line 6 Assumption 3 is discharged on line 8
2	(2)	P	A / CP ⊢ -P	
3	(3)	-P	A / RAA	
2,3	(4)	P & -P	2,3 &I	
2,3	(5)	-P	4 &E	
~~2~~,3	(6)	P → -P	2,5 CP	
1,3	(7)	-(P → -P)&(P → -P)	1,6 &I	
1,~~3~~	(8)	- - P	3,7 RAA	
1	(9)	P	8 DN	

What sequent is allegedly proven on line 9?

- (P → -P) ⊢ P

Is this sequent valid?

Yes

Briefly explain the purpose of line 4.

Contamination – for the purpose of getting '-P' based on assumption 2, in order to do the '2,5 CP' move on line 6. '2,3 CP' is illegal. When doing the CP move, the first number cited (when in numerical order) must be one of the assumptions for the second line number cited.

What are the premises for line 6? ***2,5***

What are the assumptions for line 7? ***1,3***

Are there any errors in the proof (including illegal moves, and legal but unhelpful moves)?

Yes

If you answered "yes," briefly explain what is illegal, and/or legal but unhelpful, in the proof (including any pertinent line number(s) and rule(s) broken).

Line 7 is legal but unhelpful. Line 8 is illegal. When using the RAA rule, the second line number cited (when in numerical order) must be a wff of the form B & - B. The wff on line 7 is not in this form (which is why line 7 is legal but unhelpful).

6) **(9 points)** Consider the following sequents.

X: (-P → Q) v P , -P ⊢ -P → Q

Y: P v Q , -Q ⊢ P

Z: S v -R , R ⊢ S

For each of the following pairs, state whether or not the first sequent is a substitution-instance of the second..

X-X ***Yes***	Y-X ***No***	Z-X ***No***
X-Y ***Yes***	Y-Y ***Yes***	Z-Y ***No***
X-Z ***No***	Y-Z ***No***	Z-Z ***Yes***

Extra Credit (5 points):
Prove the other sequent from question 4 above.

- (P → -Q) ├ P & Q

1	(1)	- (P → -Q)	A
2	(2)	- (P & Q)	A / RAA
3	(3)	P	A / CP ├ -Q
4	(4)	Q	A / RAA
3,4	(5)	P & Q	3,4 &I
2,3,4	(6)	(P & Q) & - (P & Q)	2,5 &I
2,3,**4**	(7)	- Q	4,6 RAA
2,~~**3**~~	(8)	P → -Q	3,7 CP
1,2	(9)	(P → -Q) & - (P → -Q)	1,8 &I
1,~~**2**~~	(10)	- - (P & Q)	2,9 RAA
1	(11)	P & Q	10 DN

(Assumption 4 is discharged on line 7)
(Assumption 3 is discharged on line 8)
(Assumption 2 is discharged on line 10)